

**University of Bahrain**  
**College of Information technology**  
**Department of Computer Engineering**

**Test (1)**

|              |                 |
|--------------|-----------------|
| Student Name |                 |
| I.D. No.     | <i>Solution</i> |
| Section      |                 |

**Course Title:** Digital Logic  
**Course number:** ITCE 202/250  
**Semester:** 1  
**Academic Year:** 2013/2014  
**Duration :** 1hour  
**Date:** 29/10/2013

**Read the following before you start:**

1. Write your name, ID and section number
2. Answer all questions.
3. Write your answers on the attached sheets only.

| Question | Mark | Mark attained |
|----------|------|---------------|
| 1        | 22   |               |
| 2        | 16   |               |
| 3        | 15   |               |
| 4        | 22   |               |
| 5        | 25   |               |
| Total    | 100  |               |

**Question [1]: [22 mark]**

(a) Convert the following numbers showing all steps.

3  $(1111)_2 = ( \quad )_{BCD} \quad 15_{10} = 0001\ 0101_{BCD}$

3  $(A29)_{16} = ( \quad )_4$   $\begin{array}{cccccc} 1010 & 0010 & 1001 & & & \\ 2 & 2 & 0 & 2 & 2 & 14 \end{array}$

3  $(15)_{10} = ( \quad )_{\text{excess}_3}$   $0001\ 0101_{BCD} \xrightarrow{+3} 0100\ 1000_{\text{excess}_3}$

3  $(-35)_{10} = ( \quad )_{1's\ complement}$

64 32 16 8 4 2 1  
Binary +35 0 1 0 0 0 1 1  
1's Comp -35 1 0 1 1 1 0 0

(b) Add the following numbers in BCD

5  $(97)_{10} + (25)_{10} =$

|      |      |     |
|------|------|-----|
| 1001 | 0111 | 97  |
| 0010 | 0101 | 25  |
| 1011 |      | 122 |
| 0110 | 0110 |     |
| 0001 |      |     |
| 0010 | 0010 |     |

5 b) Perform the following operation using 6-bit 2's complement numbers and indicate the case of an overflow.

$(-20)_{10} + (-15)_{10} =$

|     |    |    |   |   |   |   |
|-----|----|----|---|---|---|---|
|     | 32 | 16 | 8 | 4 | 2 | 1 |
| +20 | 0  | 1  | 0 | 1 | 0 | 0 |
| +15 | 0  | 0  | 1 | 1 | 1 | 1 |

|     |        |
|-----|--------|
| -20 | 101100 |
| -15 | 110001 |
| -35 | 101101 |
|     | C=1    |
|     | V=1    |

## Question [2]: [16 mark]

1. Simplify the following expression using the Boolean algebra to a minimum number of literals:

$$\overline{A+B} + ABC + A\overline{B}$$

$$\overline{A}\overline{B} + ABC + A\overline{B}$$

$$\overline{B}(\overline{A}+A) + ABC$$

$$\overline{B} + ABC$$

$$\overline{B} + AC$$

- 2- Find the minimum Product of Sum for the following function, use Boolean Algebra.

$$(a+b)(a+b+d)(a+c)$$

$$\overline{F} = \overline{a}\overline{b} + \overline{a}\overline{b}\overline{d} + \overline{a}\overline{c}$$

$$= \overline{a}\overline{b}(1+\overline{d}) + \overline{a}\overline{c}$$

$$= \overline{a}\overline{b} + \overline{a}\overline{c}$$

$$F = (a+b)(a+c)$$

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 0  | 0  |    |    |
| 01      | 0  | 0  |    |    |
| 11      | 0  |    |    |    |
| 10      | 0  |    |    |    |

$$\overline{F} = \overline{a}\overline{b} + \overline{a}\overline{c}$$

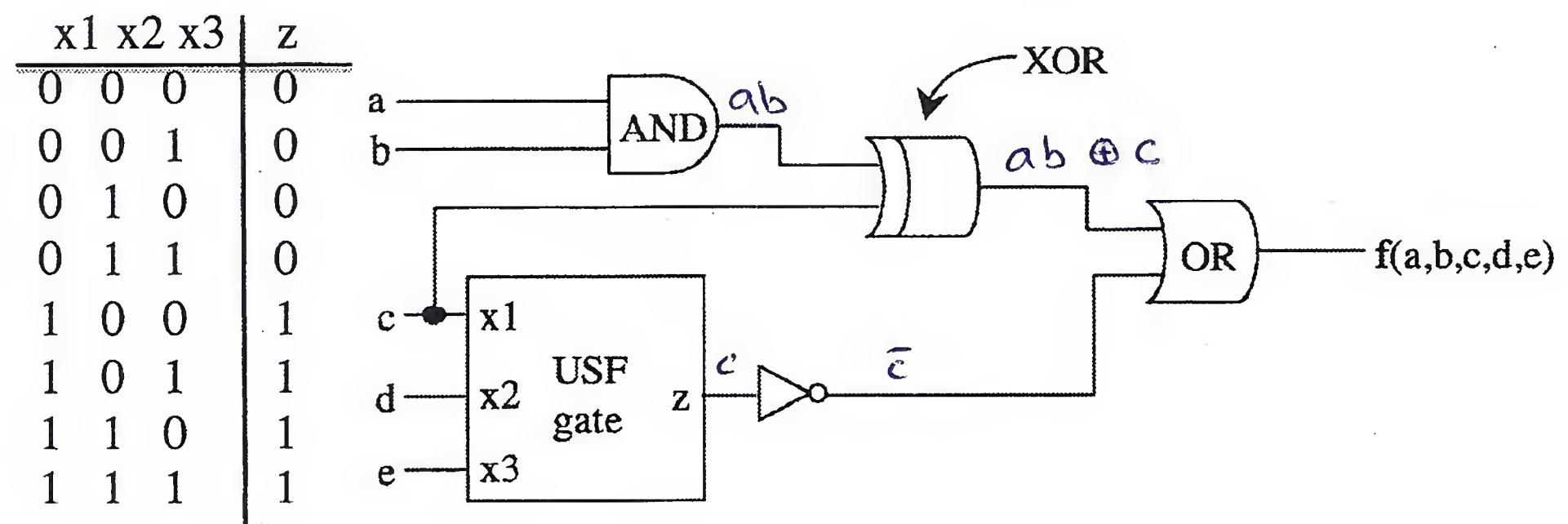
OR USE  $X \cdot (X+Y) = X$  Theorem 10D on Page 52  
to get  $(a+b)(a+b+d)(a+c)$

$$= (a+b)(a+c)$$



**Question [3]: [15 mark]**

In the given circuit below, the gate labeled “USF gate” has the truth table shown below.



Write  $f(a,b,c,d,e)$  as a minimum sum of product.

③  $z = c$

⑤  $f = (ab \oplus c) + \bar{c}$

$$= ab\bar{c} + \bar{a}\bar{b}c + \bar{c}$$

$$= ab\bar{c} + c(\bar{a} + \bar{b}) + \bar{c}$$

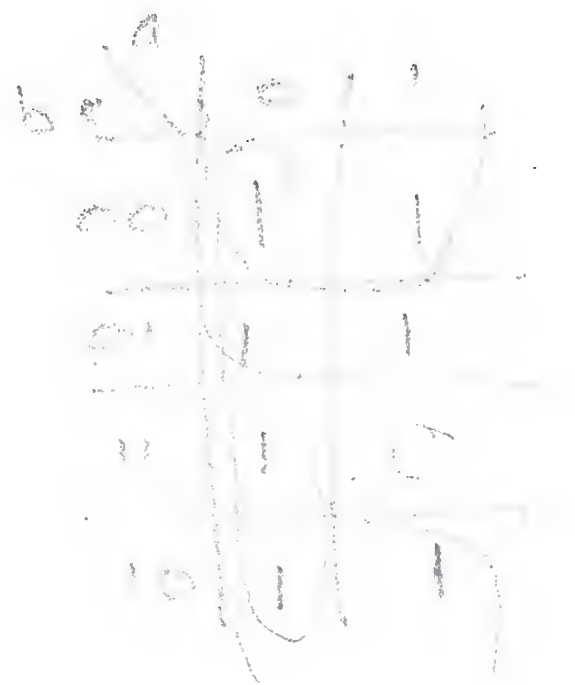
$$= ab\bar{c} + \bar{a}c + \bar{b}c + \bar{c}$$

$$= \bar{c}(ab + 1) + \bar{a}c + \bar{b}c$$

$$= \bar{c} + \bar{a}c + \bar{b}c$$

$$= \bar{c} + \bar{a} + \bar{b}c$$

$$= \bar{a} + \bar{b} + \bar{c}$$



$$\bar{a} + \bar{b} + \bar{c}$$

**Question [4]: [22 mark]**

a-- Find the maxterm expansion in algebraical expansion (in complete form) of the following expression:

$$F(X, Y, Z) = XY + \bar{X}Z + X\bar{Y}\bar{Z}$$

$$F = \sum m(1, 3, 4, 6, 7).$$

$$= \prod M(0, 2, 5).$$

$$\bar{F} = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z$$

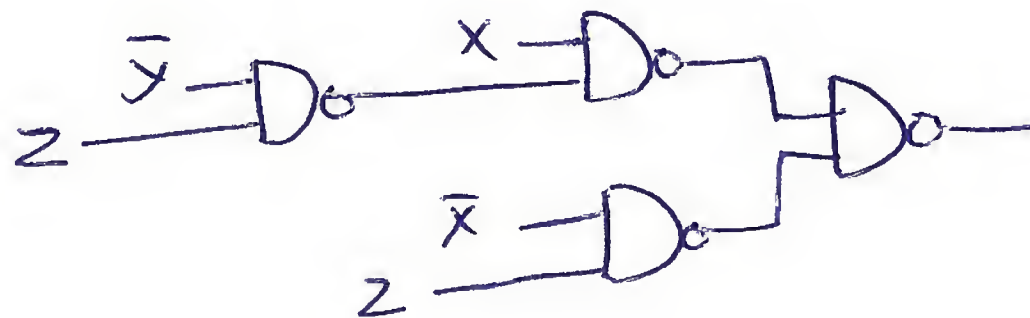
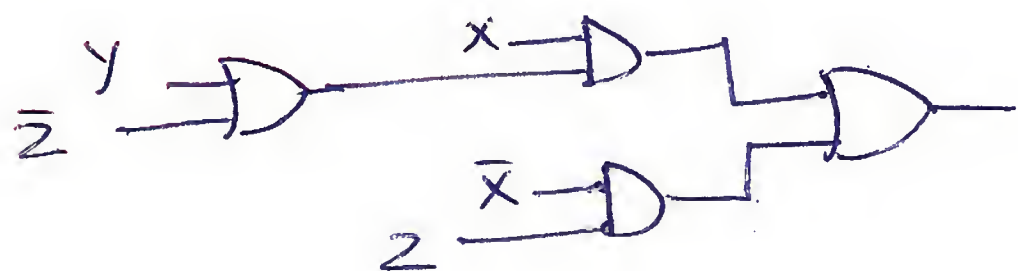
$$F = (X + Y + Z)(X + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})$$

| $Y \backslash X$ | 0 | 1 | 1 |
|------------------|---|---|---|
| 00               | 0 | 1 | 1 |
| 01               | 1 | 0 | 1 |
| 11               | 1 | 1 | 1 |
| 10               | 0 | 1 | 1 |

b-- Implement F using minimum 2-inputs NAND gates.

$$F = XY + \bar{X}Z + X\bar{Z}$$

$$= X(Y + \bar{Z}) + \bar{X}Z$$



c-- Write  $F(a, b, c, d) = \sum m(5, 6, 7, 8, 9, 10, 11, 12, 13, 15) + \sum d(0, 3, 14)$  as minimum sum of product.

$$F = a + bd + bc$$

| $cd \backslash ab$ | 00 | 01 | 11 | 10 |
|--------------------|----|----|----|----|
| 00                 | X  |    | 1  | 1  |
| 01                 |    | 1  | 1  | 1  |
| 11                 | X  | 1  | 1  | 1  |
| 10                 |    | 1  | X  | 1  |



**Question [5]: [25 mark]**

A combinational logic circuit receives BCD numbers as input. The output (W, X, Y, Z) represents the excess-3 code of the inputs. Consider the invalid inputs as don't care cases.

- (a) Construct the truth table.  
 (b) Find the minterm expansion of W in decimal notation.  
 (c) Find the maxterm expansion of X in decimal notation.  
 (d) Find the minimum SOP expression of Z.  
 (e) Implement Y using minimum number of NOR gates only.

b)  $W = \sum m(5, 6, 7, 8, 9) + \sum d(10, 11, 12, 13, 14, 15)$

c)  $X = \prod M(0, 5, 6, 7, 8) \cdot \prod D(10, 11, 12, 13, 14, 15)$

d)

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | X  | 1  |
| 01      | 0  | 0  | X  | 0  |
| 11      | 0  | 0  | X  | X  |
| 10      | 1  | 1  | X  | X  |

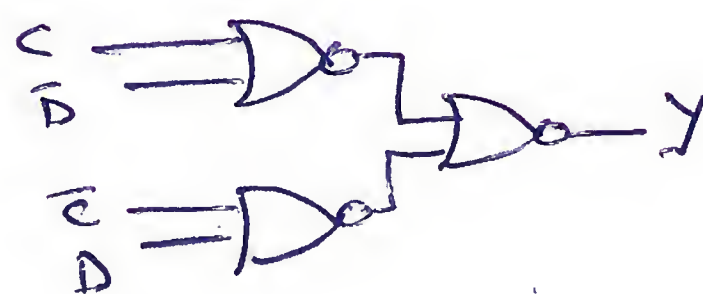
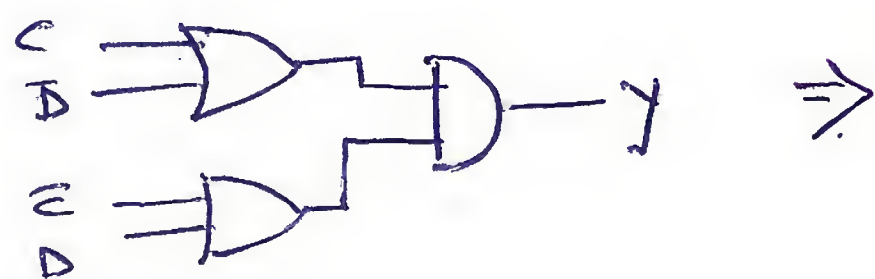
$Z = \bar{D}$

e)

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00      | 1  | 1  | X  | 1  |
| 01      | 0  | 0  | X  | 0  |
| 11      | 1  | 1  | X  | X  |
| 10      | 0  | 0  | X  | X  |

$\bar{Y} = \bar{C}D + C\bar{D}$

$Y = (C + \bar{D})(\bar{C} + D)$



a

|   | A | B | C | D | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
|   | 1 | 0 | 1 | 0 | x | x | x | x |
|   | 1 | 0 | 1 | 1 | x | x | x | x |
|   | 1 | 1 | 0 | 0 | x | x | x | x |
|   | 1 | 1 | 0 | 1 | x | x | x | x |
|   | 1 | 1 | 1 | 0 | x | x | x | x |
|   | 1 | 1 | 1 | 1 | x | x | x | x |